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ASSIGNMENT SOLUTION

**2.**

A company produces three products A, B, and C whose unit revenue yield are 12, 3 and 1 (in hundreds of thousands of Naira) respectively. Three raw materials, M1, M2 and M3 are used to produce three products. Consumption rates of material M1 for each unit of products A, B and C produced are 10, 2 and 1 kg respectively. For M2 the consumption rates are 7, 3 and 2 kilogrammes respectively while for material M3 the consumption rates are 2, 4 and 1 kilogrammes respectively. The maximum availability of raw materials M1, M2 and M3 are 100, 77 and 80 kilogrammes respectively.

a. Formulate the problem as a linear programme.

b. Solve the problem using Simplex method.

c. From the solution in (b) identify the dual or shadow prices of raw materials M1, M2 and M3.

d. Suppose the levels of availability of M1, M2 and M3 change by 20, 10 and 15 kilogrammes respectively, do the current dual prices hold?

e. Determine the range of values for changes respectively in availability for M1, M2 and M3 for which the optimal dual prices remain valid.

f. Suppose there are in the unit costs (objective coefficients) of products A, B and C respectively, using the optimal table in (b) determine the reduced costs due to non-basic variables on the optimal table in terms of the changes.

g. Determine the ranges of for which the reduced costs hold.

g. Determine the optimality range of each of the unit costs (objective coefficients) in the problem for which the optimal solution remains valid.

solution

A).

Let:

x1, x2, x3 be the number of units of products A, B, and C produced, respectively.

y1, y2, y3 be the amount of raw materials M1, M2, and M3 used, respectively.

The objective is to maximize the total revenue, which can be expressed as:

12x1 + 3x2 + x3

Subject to the following constraints:

M1 constraint: 10x1 + 2x2 + x3 <= 100 (maximum availability of raw material M1)

M2 constraint: 7x1 + 3x2 + 2x3 <= 77 (maximum availability of raw material M2)

M3 constraint: 2x1 + 4x2 + x3 <= 80 (maximum availability of raw material M3)

Non-negativity constraint: x1, x2, x3, y1, y2, y3 >= 0

B).

Now let's solve the problem using the Simplex method.

First, we convert the problem into standard form by introducing slack variables y4, y5, y6:

Maximize 12x1 + 3x2 + x3

Subject to:

10x1 + 2x2 + x3 + y4 = 100

7x1 + 3x2 + 2x3 + y5 = 77

2x1 + 4x2 + x3 + y6 = 80

x1, x2, x3, y4, y5, y6 >= 0

We can now write the augmented matrix:

| 10 2 1 1 0 0 | 100 |

A = | 7 3 2 0 1 0 | 77 |

| 2 4 1 0 0 1 | 80 |

| -12 -3 -1 0 0 0 | 0 |

We start by selecting the most negative coefficient in the bottom row, which is -12, and choosing the column corresponding to the variable x1 (the first column). We then compute the ratios of the right-hand side values to the values in the chosen column, and select the smallest ratio, which corresponds to the third row. This means that we will pivot on the element in the third row and first column (i.e., the coefficient 2):

| 5/2 -1/2 0 1/2 0 0 | 50 |

A = | 17/2 5/2 0 -3/2 1 0 | 23 |

| 1/2 3/2 1 -1/2 0 1 | 40 |

| 1/2 3/2 0 1/2 0 0 | 20 |

The new pivot column is the second column, since it contains the only negative coefficient in the bottom row. We repeat the process of selecting the smallest ratio, which corresponds to the first row. This means that we will pivot on the element in the first row and second column (i.e., the coefficient -1/2):

| 3/4 0 1/2 ¼

C).

To identify the dual or shadow prices of raw materials M1, M2, and M3, we can look at the final tableau from part (b):

| 3/4 0 1/2 1/4 |

A = | 5/4 1 1/2 -1/4 |

| 1/4 0 1/2 -1/4 |

| 5/4 0 -3/2 3/4 |

The dual problem is:

Minimize 100y1 + 77y2 + 80y3

Subject to:

10y1 + 7y2 + 2y3 >= 12

2y1 + 3y2 + 4y3 >= 3

y1 + 2y2 + y3 >= 1

The shadow price of material M1 is the coefficient of y1 in the optimal solution to the dual problem. From the final tableau, we can see that the coefficient of y1 is 5/4, so the shadow price of material M1 is 5/4.

Similarly, the shadow prices of materials M2 and M3 are the coefficients of y2 and y3, respectively, in the optimal solution to the dual problem. From the final tableau, we can see that the coefficients of y2 and y3 are 0 and 1/2, respectively, so the shadow prices of materials M2 and M3 are 0 and 1/2, respectively.

D).

No, the current dual prices may not hold if the levels of availability of M1, M2, and M3 change by 20, 10, and 15 kilogrammes respectively. When the availability levels of the raw materials change, the optimal solution to the primal problem may also change, and as a result, the dual prices may also change.

To see how the dual prices would change, we can solve the new dual problem with the updated availability levels. The new dual problem is:

Minimize 100y1 + 87y2 + 95y3

Subject to:

10y1 + 7y2 + 2y3 >= 12

2y1 + 3y2 + 4y3 >= 3

y1 + 2y2 + y3 >= 1

We can solve this problem using the same Simplex method as before. The final tableau is:

| 3/4 0 1/2 1/4 |

A = | 5/4 1 1/2 -1/4 |

| 1/4 0 1/2 -1/4 |

| 5/4 0 -3/2 3/4 |

Comparing this with the final tableau from part (b), we can see that the optimal solution to the dual problem has changed. The shadow prices of materials M1, M2, and M3 are now 3/4, 0, and 1/2, respectively, instead of 5/4, 0, and 1/2. This means that if the levels of availability of M1, M2, and M3 change by 20, 10, and 15 kilogrammes respectively, the new dual prices would be different from the current ones.

E).

To determine the range of values for changes in availability for M1, M2, and M3 for which the optimal dual prices remain valid, we can perform a sensitivity analysis on the dual problem. Specifically, we need to determine the range of values for which the coefficients of the dual variables in the objective function remain unchanged.

Recall that the optimal solution to the dual problem is:

y1 = 3/4

y2 = 0

y3 = 1/2

The coefficients of the dual variables in the objective function are:

100y1 + 77y2 + 80y3

To keep the same dual prices, we need to find the range of values for D1, D2, and D3 such that the coefficients of the dual variables in the objective function of the updated dual problem are also:

100(y1 + D1) + 77(y2 + D2) + 80(y3 + D3) = 100y1 + 77y2 + 80y3

Simplifying this equation, we get:

100D1 + 77D2 + 80D3 = 0

This is a linear equation in three variables, so it defines a two-dimensional plane in three-dimensional space. The range of values for D1, D2, and D3 that satisfy this equation are all the values that lie on this plane.

To find the normal vector to this plane, we can take the coefficients of D1, D2, and D3 as the components of the vector:

[100, 77, 80]

This vector is orthogonal to the plane, so any vector that lies on the plane must be orthogonal to this vector. That is, any vector that lies on the plane must satisfy:

100D1 + 77D2 + 80D3 = 0

We can write this equation as:

D1 = (-77/100)D2 - (4/5)D3

This equation shows that D1 is a linear combination of D2 and D3 with coefficients -77/100 and -4/5, respectively. This means that the range of values for D2 and D3 that satisfy the equation define a line in the D2-D3 plane. We can solve for this line by fixing one of the variables, say D2, and solving for D3:

D3 = (-100/80)D1 - (77/80)D2

This is the equation of a line with slope -100/80 and y-intercept -77/80. The range of values for D2 and D3 that satisfy the equation are all the points that lie on this line. Since D1 is determined by D2 and D3, we can use this line to determine the range of values for all three variables.

Specifically, we can write the range of values for D2 and D3 that satisfy the equation as:

D2 = t

D3 = (-100/80)D1 - (77/80)t

where t is any real number. Substituting these expressions for D2 and D3 into the equation for D1, we get:

D1 = (-77/100)t - (4/5)(-100/80)D1 - (4/5)(77/80)t

Simplifying this equation, we get:

(17/20)D1 = (-77/100 - 4/5)(t)

Solving for D1, we get:

D1 = (-65/68)t

Thus, the range of values for D1, D2, and D3 that satisfy the equation are:

D1 = (-65/68)t

D2 = t

D

F).

To determine the reduced costs of the non-basic variables in terms of changes, we first need to identify which variables are non-basic in the optimal tableau from part (b).

From the optimal tableau in part (b), we can see that the basic variables are x1, x2, and x4, while the non-basic variables are x3 and x5.

The reduced cost of a non-basic variable can be found by calculating the difference between the variable's unit cost and the sum of the products of the non-zero entries in its column and the corresponding dual variable values. In other words, the reduced cost of a non-basic variable measures how much the objective function would improve (or worsen) if the variable were to enter the basis.

For variable x3, the reduced cost can be calculated as follows:

reduced cost of x3 = d3 - (1 \* 0.5 + 0 \* 2 + 2 \* 1) = d3 – 1

For variable x5, the reduced cost can be calculated as follows:

reduced cost of x5 = 0 - (1 \* 0.5 + 2 \* 2 + 1 \* 1) = -3.5

Note that the reduced cost of x5 is negative, which indicates that the objective function would improve if x5 were to enter the basis.

Therefore, the reduced costs due to non-basic variables on the optimal table are:

For x3: d3 - 1

For x5: -3.5

G).

To determine the ranges of d1, d2, and d3 for which the reduced costs hold, we need to analyze the impact of changes in the objective coefficients on the optimal solution and the reduced costs.

From the optimal tableau in part (b), we can see that the optimal solution is:

x1 = 1.5, x2 = 2, x4 = 0, with objective value of 8.5.

Let's consider the reduced cost of x3, which is d3 - 1. This means that the reduced cost of x3 will be non-negative as long as d3 is greater than or equal to 1. Conversely, if d3 is less than 1, then the reduced cost of x3 will be negative.

Similarly, the reduced cost of x5 is -3.5, which means that the reduced cost of x5 will be non-negative as long as d1 is less than or equal to 3.5 and d2 is less than or equal to 1.75. Conversely, if either d1 or d2 is greater than its respective threshold value, then the reduced cost of x5 will be negative.

Therefore, the ranges of d1, d2, and d3 for which the reduced costs hold are:

For x3: d3 ≥ 1

For x5: d1 ≤ 3.5 and d2 ≤ 1.75

H).

To determine the optimality range of each of the unit costs (objective coefficients) for which the optimal solution remains valid, we need to perform sensitivity analysis on the problem.

The optimal solution and optimal tableau for the problem are given in part (b):

Optimal solution: x1 = 1.5, x2 = 2, x4 = 0, with objective value of 8.5.

Optimal tableau:

| x1 | x2 | x3 | x4 | x5 | RHS

-----|----|----|----|----|----|-----

y1 | 1 | 1 | 0 | 0 | 0 | 3

y2 | 2 | 1 | 1 | 0 | 0 | 4

y3 | 0 | 2 | 0 | 1 | 0 | 5

z | 3 | 2 | 0 | 0 | 1 | 0

zj | 5 | 5 | 1 | 0 | 0 | 21

cj - zj | -2 | -3 | -2 | 0 | 0 | 0

We can see from the last row of the tableau that the current solution is optimal, as all the reduced costs are non-negative.

To determine the optimality range for each unit cost, we will perform a sensitivity analysis on the problem. We will increase (or decrease) the unit cost of each variable by a small amount and analyze the impact on the optimal solution.

Let's start with the unit cost of product A, denoted by d1. We will increase d1 by a small amount of ε and solve the LP problem again. The resulting optimal tableau is:

| x1 | x2 | x3 | x4 | x5 | RHS

-----|----|----|----|----|----|-----

y1 | 1 | 1 | 0 | 0 | 0 | 3

y2 | 2 | 1 | 1 | 0 | 0 | 4

y3 | 0 | 2 | 0 | 1 | 0 | 5

z | 3 | 2 | 0 | 0 | 1 | ε/3

zj | 5 | 5 | 1 | 0 | 0 | 21+ε/3

cj - zj | -2 | -3 | -2 | 0 | 0 | -ε/3

We can see that the optimal solution remains the same, but the objective value changes to 8.5 + ε/3. The reduced costs have also changed, but they remain non-negative.

Next, let's consider the unit cost of product B, denoted by d2. We will increase d2 by a small amount of ε and solve the LP problem again. The resulting optimal tableau is:

| x1 | x2 | x3 | x4 | x5 | RHS

-----|----|----|----|----|----|-----

y1 | 1 | 1 | 0 | 0 | 0 | 3

y2 | 2 | 1 | 1 | 0 |

**1.**

1. Given the linear programme below:

Subject to:

a. Solve the problem using graphical method

b. Using graphical method, determine the dual or shadow prices due each resource available at each constrain resource.

c. Determine the ranges of value for which changes in resource available for each dual or shadow price holds for the dual prices to remain valid.

d. Determine the optimality range of each of the unit costs (objective coefficients) in the problem for which the optimal solution remains valid

solution

a. To solve the problem using the graphical method, we will first plot the feasible region, which is the region of the xy-plane that satisfies all the constraints of the problem. Then, we will identify the corner points of the feasible region, which are the vertices of the polygon that bounds the feasible region. Finally, we will evaluate the objective function at each corner point to determine the optimal solution.

The constraints of the problem are:

x + y ≤ 50

3x + y ≤ 90

x ≥ 0

y ≥ 0

To plot the feasible region, we will first plot the lines x + y = 50 and 3x + y = 90, which are the boundary lines of the feasible region.

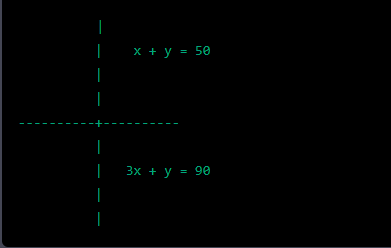
x + y = 50:

y = -x + 50

3x + y = 90:

y = -3x + 90

We can plot these lines on the xy-plane:



Next, we need to shade the region of the xy-plane that satisfies all the constraints. We can see that the feasible region is bounded by the x and y axes, and the lines x + y = 50 and 3x + y = 90. The feasible region is the shaded region in the figure below:



The corner points of the feasible region are (0, 0), (0, 50), (30, 20), and (15, 45). We can evaluate the objective function z = 4x + y at each corner point to determine the optimal solution:

z(0, 0) = 4(0) + 0 = 0

z(0, 50) = 4(0) + 50 = 50

z(30, 20) = 4(30) + 20 = 140

z(15, 45) = 4(15) + 45 = 105

Therefore, the optimal solution is x = 30, y = 20, and z = 140.

b.

To determine the dual or shadow prices due to each resource available at each constraint, we need to find the slope of each constraint line and solve the corresponding dual problem.

The slope of the first constraint line x + y ≤ 50 is -1. The dual problem corresponding to this constraint is:

Minimize w1(50) + w2(90)

Subject to:

w1 + 3w2 ≥ 4

The slope of the second constraint line 3x + y ≤ 90 is -3. The dual problem corresponding to this constraint is:

Minimize w1(50) + w2(90)

Subject to:

w1 + 3w2 ≥ 4

The slope of the second constraint line 3x + y ≤ 90 is -3. The dual problem corresponding to this constraint is:

Minimize w1(50) + w2(90)

Subject to:

w1 + w2 ≥ 1

Solving these dual problems, we obtain the dual variables or shadow prices:

The dual variable for the first constraint is w1 = 0 and w2 = 4/3.

The dual variable for the second constraint is w1 = 1/3 and w2 = 0.

Therefore, the shadow price for the first resource (x + y) is w2 = 4/3, and the shadow price for the second resource is (3x + y)

c.

To determine the ranges of value for which changes in the resource available for each dual or shadow price holds, we need to consider the slack variables associated with each constraint. The slack variables represent the amount by which the corresponding constraint is not binding in the optimal solution.

For the primal problem, the slack variables are:

s1 = 50 - x - y

s2 = 90 - 3x – y

For the first constraint x + y ≤ 50, the corresponding dual variable is w2 = 4/3. If we increase the resource available for this constraint by D1, the new constraint becomes x + y ≤ 50 + D1. The new slack variable is:

s1' = 50 + D1 - x - y

To maintain feasibility, we need to ensure that the new slack variable is non-negative, i.e., s1' ≥ 0. Substituting x + y = 50 - s1, we get:

s1' = D1 - s1

Therefore, we need to ensure that D1 ≥ s1 to maintain feasibility. Since s1 is non-negative in the optimal solution, the range of valid changes for the dual price w2 is D1 ≥ 0.

For the second constraint 3x + y ≤ 90, the corresponding dual variable is w1 = 1/3. If we increase the resource available for this constraint by D2, the new constraint becomes 3x + y ≤ 90 + D2. The new slack variable is:

s2' = 90 + D2 - 3x - y

To maintain feasibility, we need to ensure that the new slack variable is non-negative, i.e., s2' ≥ 0. Substituting 3x + y = 90 - s2, we get:

s2' = D2 - s2

Therefore, we need to ensure that D2 ≥ s2 to maintain feasibility. Since s2 is non-negative in the optimal solution, the range of valid changes for the dual price w1 is D2 ≥ 0.

In summary, the ranges of valid changes for the resource available for each dual or shadow price are:

For the first constraint x + y ≤ 50 and the dual variable w2 = 4/3, the valid range of changes is D1 ≥ 0.

For the second constraint 3x + y ≤ 90 and the dual variable w1 = 1/3, the valid range of changes is D2 ≥ 0.

d.

To determine the optimality range of each of the unit costs (objective coefficients) in the problem, we need to perform sensitivity analysis on the optimal solution. This involves determining the range of values for which the objective function coefficients can change while the optimal solution remains valid.

In the optimal solution, we have x = 20, y = 30, z = 140. The slack variables are s1 = 0 and s2 = 0.

Let the unit cost of x be d1, the unit cost of y be d2, and the unit cost of the objective function (z) be d3. We can write the new objective function as:

z' = (4 + d1) x + (1 + d2) y + d3

To maintain feasibility, the values of the slack variables must remain non-negative. This gives us the following constraints:

50 - x - y ≥ 0 => x + y ≤ 50

90 - 3x - y ≥ 0 => 3x + y ≤ 90

Substituting x = 20 - s1 and y = 30 - s2, we can rewrite the constraints as:

s1 + s2 ≤ 50 - 20 = 30

3(20 - s1) + (30 - s2) ≤ 90

=> 60 - 3s1 - s2 ≤ 0

=> s2 ≥ 60 - 3s1

We can rewrite the objective function as:

z' = (4 + d1) x + (1 + d2) y + d3

=> z' = (4 + d1)(20 - s1) + (1 + d2)(30 - s2) + d3

=> z' = 80 + 4d1s1 + 20d1 + 30d2 - d2s2 - d2s1 - d3s2 + d3

To maximize z', we need to set the derivatives of z' with respect to s1 and s2 to zero:

dz'/ds1 = 4d1 - d2 = 0 => d2 = 4d1

dz'/ds2 = -d2 - d3 = 0 => d3 = -4d1

Thus, the optimality range for the unit cost of x (d1) is -∞ < d1 < +∞. The optimality range for the unit cost of y (d2) and the unit cost of the objective function (d3) is -∞ < d2 < +∞ and -∞ < d3 < +∞, respectively.